1b) $e^{3x+4y-15} = 2$

$3x+4y-15 = \ln 2$ ... obecná rce přím.

1c) $\sin(x^2+y^2-25) = f(x,y)$

vrstevnice obsahující $\left(7/2, \frac{7\sqrt{3}}{2}\right) = A$

$$\sqrt{\left(\frac{7}{2}\right)^2+\left(\frac{7\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}(49+3\cdot 49)} = 7$$

$$A \in \{(x,y)\in\mathbb{R}^2 : x^2+y^2=7^2\}$$

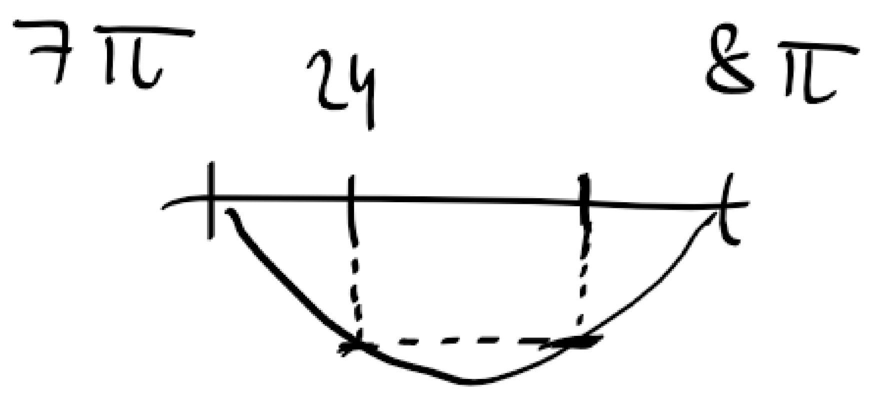
$$f(A) = f\left(\frac{7}{2}, \frac{7\sqrt{3}}{2}\right) = \sin(49-25) = \sin(24).$$

Vrstevnice je množina $= V :=$

$$:= \{(x,y)\in\mathbb{R}^2 : \sin(x^2+y^2-25) = \sin(24)\}$$

$24 \in (7\pi, 8\pi)$

$\underbrace{\sin(\alpha) = \sin(24)}_{(*)}$



$V = \{(x,y)\in\mathbb{R}^2 : x^2+y^2-25 = \alpha$, kde α je řešení lib. $(*)\}$

1g) $f(x,y,z) = \underbrace{x+y+z}_{} = c$

C – vrstevnice – rovnice roviny

1h)

3a) $16y^2 - 4x + 10 = f(x,y)$

$\varphi(t) = (0,0) + t\cdot(1,1) = (0+t\cdot 1, 0+t\cdot 1) = (t,t)$

$\varphi : \mathbb{R} \to \mathbb{R}^2$ ... křivky

$f\circ\varphi(t) = f(\varphi(t)) = f(t,t) =$

$= 16t^2 - 4t + 10$

$(f\circ\varphi)'(t) = (16t^2 - 4t + 10)' = 32t - 4$

$\varphi(t) = (\cos t, 3\sin t) \quad t \in [0, 2\pi]$

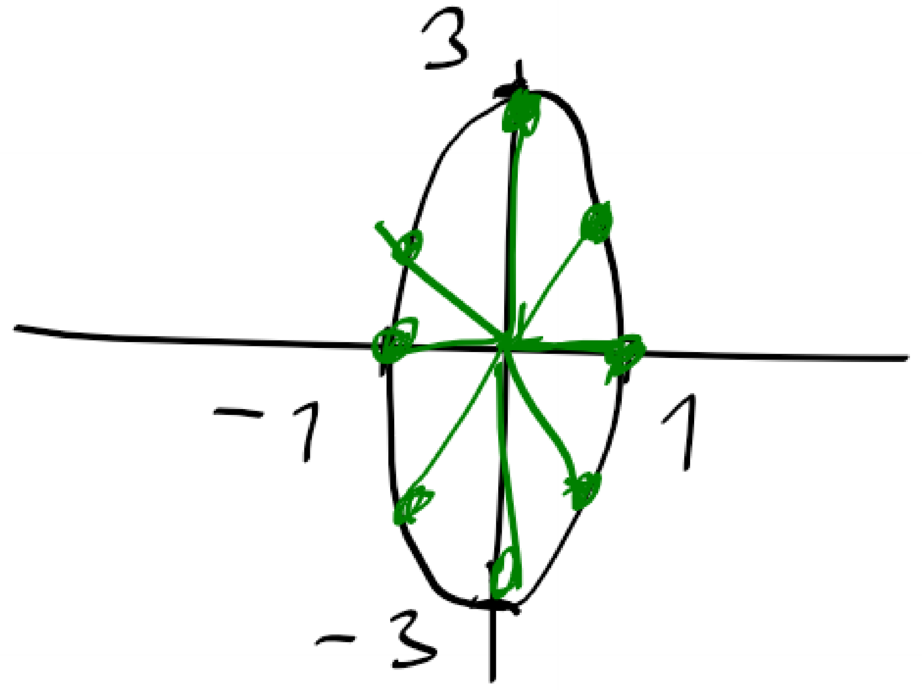
$g(x,y) = 9x^2 + y^2 \Rightarrow$

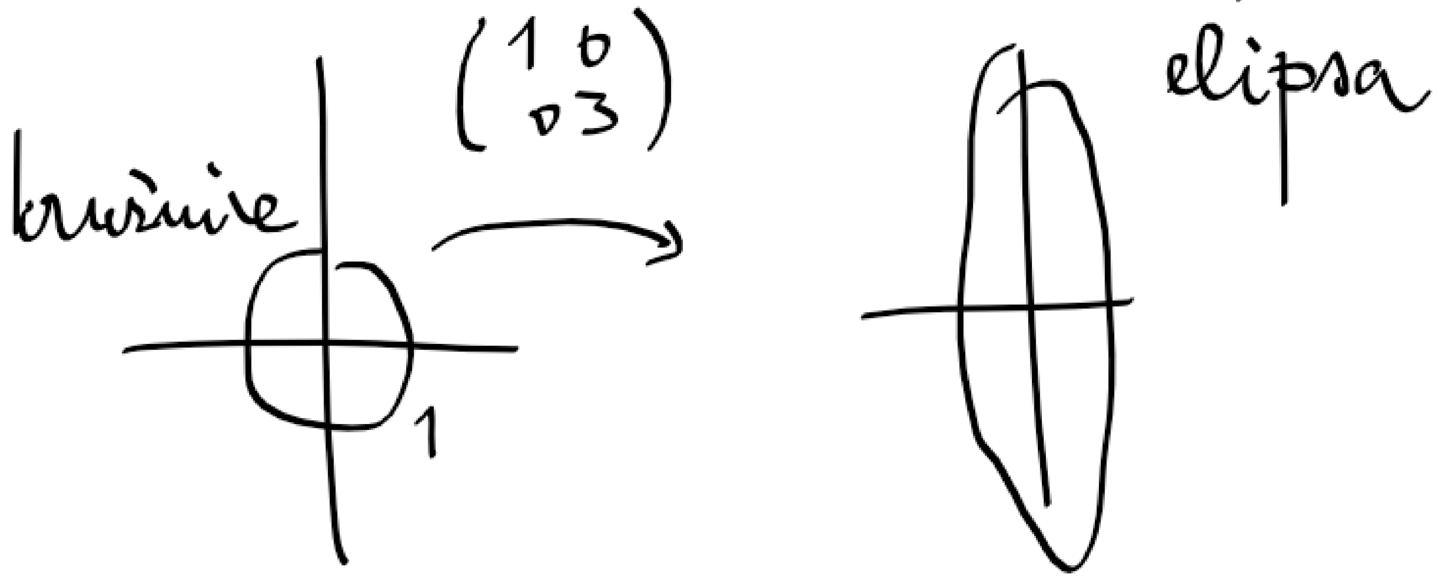
$g(\varphi(t)) = 9\cos^2 t + 9\sin^2 t = 9$

$\varphi(t) = (\underbrace{x(t)}_{\cos t}, \underbrace{y(t)}_{3\sin t})$ a platí $\forall t$:

$9x(t)^2 + y(t)^2 = 9$ tj.

$9x^2 + y^2 = 9$ rovnice elipsy





$f(x,y,z) = xyz \qquad p(t) = (20-t,\, t,\, 20)$

$f\circ p(t) = (20-t)\cdot t\cdot 20 = 20\cdot(20t - t^2)$

$(f\circ p)'(t) = 20\cdot(20-2t) = 0 \Rightarrow t = 10.$

$(f\circ p)''(t) = 20\cdot(-2) = -40 < 0$

Tedy $f\circ p$ je konk. v $t=10 \Rightarrow$ maximum.

$f\circ p(10) = 20\,(20\cdot 10 - 10^2) = 2000$

$t=10$ odpovídá bodu $p(t) = p(10) =$

$= (20-10,\, 10,\, 20) = (10,10,20) \in \mathbb{R}^3$

$$\frac{x^2 - y^2 - y}{\underbrace{(x-y)}_{x=y}\underbrace{(x-2)}_{x=2}\underbrace{(y+1)}_{y=-1}} = f(x,y)$$

$$\lim_{(x,y)\to(3,2)} f(x,y) = f(3,2) = \frac{3^2-2^2-2}{(3-2)(3-2)(2+1)}$$